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Radiative characteristics of fibers with a large size parameter

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Abstract

This study shows that the scattering intensity by a single fiber having a large size parameter and a large absorption index can be approximated by superimposition of the diffraction intensity by a rectangular aperture and the specular reflection intensity off the fiber surface. Using this approximation, the method for determining the radiative characteristics have been derived. By extending this method, we have also derived the radiative characteristics of fibers having a diffuse surface. Furthermore, we also use the resultant radiative characteristics to estimate the radiative properties of media having randomly oriented fibers and radiation transfer in the media.

1. Introduction

Fibrous media have recently been applied for use in a radiation burner [1] and solar energy collector [2] due to their ability to convert energy between thermal radiation and gas enthalpy. In such energy conversion systems, since radiation is emitted from high-temperature sources, its wavelength is usually less than the medium's fiber diameter. Thus, evaluation of the radiation transfer in fibrous media having a large size parameter is essential to improve system performance.

When the wavelength is longer than the fiber diameter, the radiative characteristics of the fiber does not depend on its surface roughness because the roughness is much smaller than the wavelength. However, as the wavelength shortens, the effect of the roughness on the radiative transfer cannot be neglected. Therefore, in fibrous media with a large size parameter, it is also important to clarify the effect of surface characteristics on the radiative transfer.

To adequately evaluate the radiation transfer in a medium of fibers with a large size parameter, a model that establishes the relationship between the radiative characteristics of individual fibers and the radiative transfer is required. In this study, we use a pseudo-continuous model, which can clarify the effect of the fiber orientation as well as the radiative characteristics of fibers on the radiation transfer.

Several pseudo-continuous models [3–6] have been developed to clarify the effect of the fiber's radiative characteristics and orientation on the radiation transfer, yet they have not been applied to media with a size parameter greater than 10. These media have only been studied to evaluate their thermal insulation characteristics at comparatively low-temperature radiation transfer conditions.

In such models, the radiative characteristics of a

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Nomenclature

d	fiber diameter, m
f_v	volume fraction of fibrous medium
I	energy of scattering radiation, W/(rad m)
i	intensity, W/(m^2 sr)
i'_{aper}	radiation energy diffracted by a rectangular aperture, W/sr
i_0	radiation heat flux, W/m^2
L	thickness of medium, m
m	complex refractive index ($= n - j\kappa$)
n	refractive index (real part of complex refractive index)
P	probability density function for fiber orientation, sr^{-1}
Q	efficiency
R_{nh}	hemispherical reflectance
T_{nh}	hemispherical transmittance
α	size parameter ($= \pi d/\lambda$)
β	extinction coefficient, m^{-1}
Φ	scattering phase function of fibrous media
κ	absorption index (imaginary part of complex refractive index)
λ	wavelength of radiation in vacuum, m
θ', θ, ϕ	coordinates in planar media (Fig. 11)
ρ	specular reflectivity off surface
ρ_{d}	diffuse reflectivity off surface
σ_{s}	scattering coefficient, m^{-1}
ω	albedo
ξ_0, ξ, η	fiber coordinates (Fig. 1)

Subscripts

abs	absorption
drefl	diffuse reflection
dif	diffraction
ext	extinction
refl	(specular) reflection
sca	scattering

single fiber have usually been evaluated using analytical solutions of Maxwell equations for electromagnetic scattering by a single fiber. The solution is composed of infinite series functions employing integral-order Bessel and Hankel functions. For a single fiber having a large size parameter and a large absorption index, it is difficult to calculate its radiative characteristics using the analytical solution. This is another reason why the pseudo-continuous models have not been applied to media with a large size parameter.

Recently, this difficulty has been overcome by Swathi and Tong [7]. They present a useful algorithm for computing the analytical solution for the radiative characteristics of a single fiber having a large size parameter and a large absorption index. In their method, the Bessel and Hankel functions in the analytical solution are scaled by exponential functions, that is, this algorithm is established on the basis of mathematical approximation.

We also present a method for determining the radiative characteristics of a single fiber in order to evaluate the radiative transfer in media of fibers with a large size parameter. The method is developed on the basis of a physical approximation for the scattering phenomenon by fibers. This study shows that the scattering intensity by a single fiber having a large size parameter and a large absorption index can be approximated by superimposition of the diffraction intensity by a rectangular aperture and the specular reflection intensity off the fiber surface. Using this approximation, the method for determining the radiative characteristics has been derived.

By extending this method, we have also derived a method for determining the radiative characteristics of fibers having a diffuse surface, assuming that the scattering is caused by diffraction through a rectangular aperture and diffuse reflection off the fiber surface.

Furthermore, we also use the resultant radiative characteristics to estimate the radiative properties of media having randomly oriented fibers and radiation transfer in the media. Based on our results that cover a wide range of size parameters, we subsequently dis-

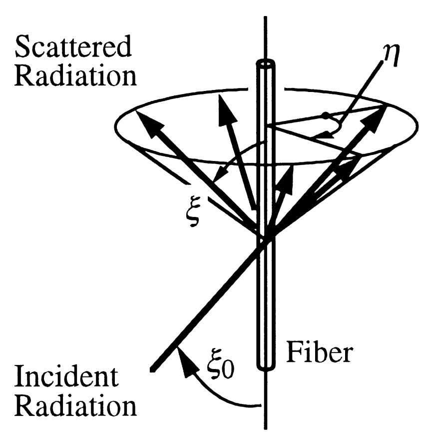

Fig. 1. Scattering by a single fiber.

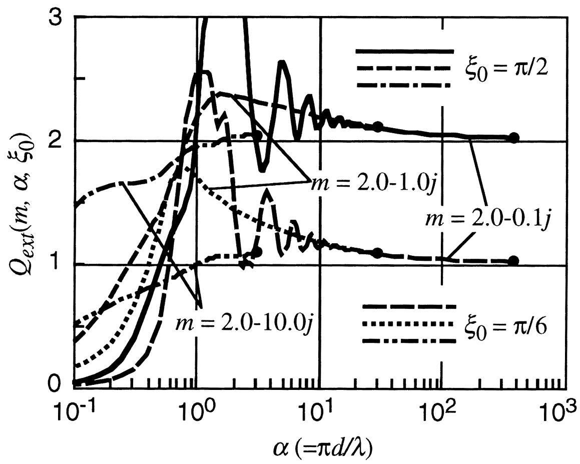

Fig. 2. Analytical solution for extinction efficiency versus size parameter at various values of κ and ξ_0.

cuss the effects of the size parameter and the surface characteristics on the radiative properties and radiation transfer.

2. Radiative characteristics of a single fiber

Fig. 1 schematically shows how an infinitely long fiber scatters radiation when it is irradiated by collimated radiation, where the radiation is scattered along the surface of a cone having a cone angle of $2\xi_0$ [8,9]. We express this type of scattering using a fiber's radiative characteristics, i.e., its intensity function and extinction and scattering efficiencies.

2.1. Analytical solutions for fiber radiative characteristics

The analytical solution for determining the radiative characteristics of a single fiber with a circular cross section is well-known (e.g., see Refs. [8,9]), with the extinction and scattering efficiencies for unpolarized radiation being expressed as functions of the fiber diameter d, the wavelength λ of radiation in a vacuum, and the material's complex refractive index m, i.e.,

$$Q_{\text{ext}}(m, \alpha, \xi_0) = \frac{2}{\alpha} Re\big[T(\eta = 0)\big] \tag{1}$$

$$Q_{\text{sca}}(m, \alpha, \xi_0) = \frac{1}{\pi\alpha}\int_0^{2\pi} i(\xi_0, \eta)\, \mathrm{d}\eta \tag{2}$$

where $T(\eta = 0)$ and $i(\xi_0, \eta)$ are respectively the amplitude and intensity functions, which are infinite series functions employing integral-order Bessel and Neumann functions (for details see Kerker [9]).

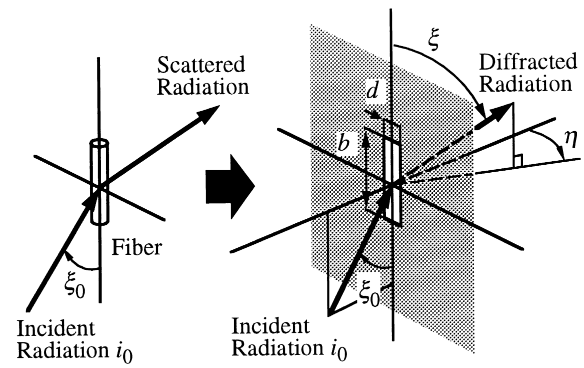

Fig. 3. Rectangular aperture model for representing diffraction in a fiber.

Eq. (1) enables the extinction efficiency Q_{ext} to be numerically calculated at single precision. Fig. 2 shows its resultant dependency on the size parameter $\alpha\,(= \pi d/\lambda)$ at indicated values of m and the angle between the incident radiation and fiber axis, ξ_0 (Fig. 1). The upper limits of the extinction coefficients are also shown. It is important to note that the analytical solution cannot determine Q_{ext} at large values of α, and as the absorption index κ increases, the upper limit is reduced.

The exact solution reaches an upper limit during the calculation because the constant in the Bessel function exceeds its allowed limit imposed by the computational system. The algorithm presented by Swathi and Tong accounts for this problem. In the following, we will show another method for determining the radiative characteristics of fibers with large α and κ, and present its applicability to fibers having a diffuse surface, which is not produced from the Swathi and Tong's method using mathematical approximations.

3. Physical approximation by scattering from a fiber

Radiation scattering is caused by diffraction and reflection of the incident radiation for a fiber with a large size parameter and a large absorption index. The question addressed here is whether the scattered radiation can be approximated by (i) representing the diffraction as a Fraunhofer diffraction occurring in a rectangular aperture, which is considered to be the fiber, and (ii) representing the reflection as a specular reflection off the surface of the fiber.

In essence, it is necessary to determine whether or not the superimposition of the diffraction and reflection fields can approximate the scattering electromagnetic field. However, when the fiber diameter is much larger than the wavelength, the interference by the superimposition of both fields is considered to be negligible. In this study, the superimposition of the diffraction and reflection intensities and its resultant intensity has been compared with the analytical solutions.

3.1. Scattering by diffraction

We consider that a fiber with diameter d and length b can be represented using a rectangular aperture whose width and height are respectively the same size (see Fig. 3). We can then derive the radiation energy diffracted by the fiber using the Fraunhofer diffraction in a rectangular aperture.

When a rectangular aperture is irradiated with incident radiation having an energy flux i_0, the diffracted radiation energy per unit solid angle in the direction (ξ_0, ξ, η), $i'_{\rm aper}$, is expressed as (see Appendix A)

$$i'_{\rm aper}(\xi_0, \xi, \eta) = \frac{b^2 d^2}{\lambda^2} G^2 \left(\frac{\sin\phi}{\phi}\right)^2 \left(\frac{\sin\psi}{\psi}\right)^2 i_0 \tag{3}$$

where

$$G = \frac{\sin\xi_0 + \sin\xi\cos\eta}{2} \tag{4}$$

$$\phi = (\pi b/\lambda)(\cos\xi_0 - \cos\xi) \tag{5}$$

$$\psi = (\pi d/\lambda)(-\sin\xi\sin\eta) \tag{6}$$

To represent a fiber, the rectangular aperture is assumed to be infinitely long; thus $\pi b/\lambda$ is very large such that $(\sin\phi/\phi)^2 = 0$, except when $\phi = 0$. Accordingly, most of radiation energy is scattered into direction ξ, which satisfies $\cos\xi_0 - \cos\xi = 0$, i.e., $\xi = \xi_0$. This condition represents the incident radiation on a fiber that is scattered into a cone-shaped surface having a cone angle of $2\xi_0$ (see Fig. 1).

The diffracted radiation energy in the direction $(\xi = \xi_0, \eta = \eta)$, $I_{\rm dif}(\xi_0, \eta)$, per unit azimuthal angle and per unit fiber length is obtained by integrating Eq. (3) from 0 to π with respect to ξ, and then dividing by fiber length b, i.e.,

$$I_{\rm dif}(\xi_0, \eta) = \frac{1}{b}\int_0^{2\pi} i'_{\rm aper}(\xi_0, \xi, \eta)\sin\xi \, d\xi \tag{7}$$

Considering $b/\lambda \gg 0$ and performing the integration gives

$$I_{\rm dif}(\xi_0, \eta) = \frac{\lambda}{\pi^2}\alpha^2 \sin^2\xi_0 \left(\frac{1+\cos\eta}{2}\right)^2 \left(\frac{\sin\psi'}{\psi'}\right)^2 i_0 \tag{8}$$

where

$$\psi' = (\pi d/\lambda)(-\sin\xi_0\sin\eta) \tag{9}$$

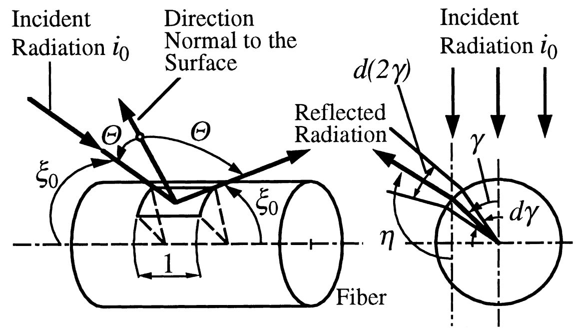

Fig. 4. Specular reflection by a fiber surface.

3.2. Scattering by reflection

Fig. 4 shows the model used to represent the specular reflection off the surface of a fiber, where the incident radiation is scattered in the direction $\xi = \xi_0$. The scattered energy $I_{\rm refl}$, which is reflected in the direction $(\xi = \xi_0, \eta = \eta)$ by area $rd\gamma \times 1$ (see Fig. 4), is expressed as

$$\rho(\Theta)\cos\gamma\sin\xi_0(rd\gamma \times 1)i_0 \tag{10}$$

where i_0 is the energy flux of the incident radiation, $\rho(\Theta)$ the reflectance off the specular surface, and Θ the angle between the incident radiation and a line normal to the fiber surface, with Θ being obtained from

$$\cos\Theta = \cos\gamma\sin\xi_0 \tag{11}$$

We calculate $\rho(\Theta)$ using the Fresnel equation [10] applied to unpolarized radiation energy traveling through air after hitting the surface of a medium having $m = n - j\kappa$.

As shown in Fig. 4, the radiation reflected off area $rd\gamma \times 1$ travels in direction $(\xi = \xi_0, \eta = \eta)$ at an angle of $d\eta$. Since $d\eta = d(2\gamma)$ and $\eta = \pi - 2\gamma$, the reflected energy per unit azimuthal angle can be represented as

$$I_{\rm refl}(\xi_0, \eta) = d/4 \times \rho(\Theta)\cos\{(\pi - \eta)/2\}\sin\xi_0 \times i_0 \tag{12}$$

3.3. Comparison between the present approximation and the analytical solutions

We approximated the radiation energy scattered per unit length of a fiber with sufficiently large α and κ values. By using the sum of the estimated diffracted and reflected energies, we compared the approximate energy distribution with the theoretical results determined using the corresponding intensity function (the integrand in Eq. (2)) derived under the electromagnetic theory. The approximate scattering energy is expressed by

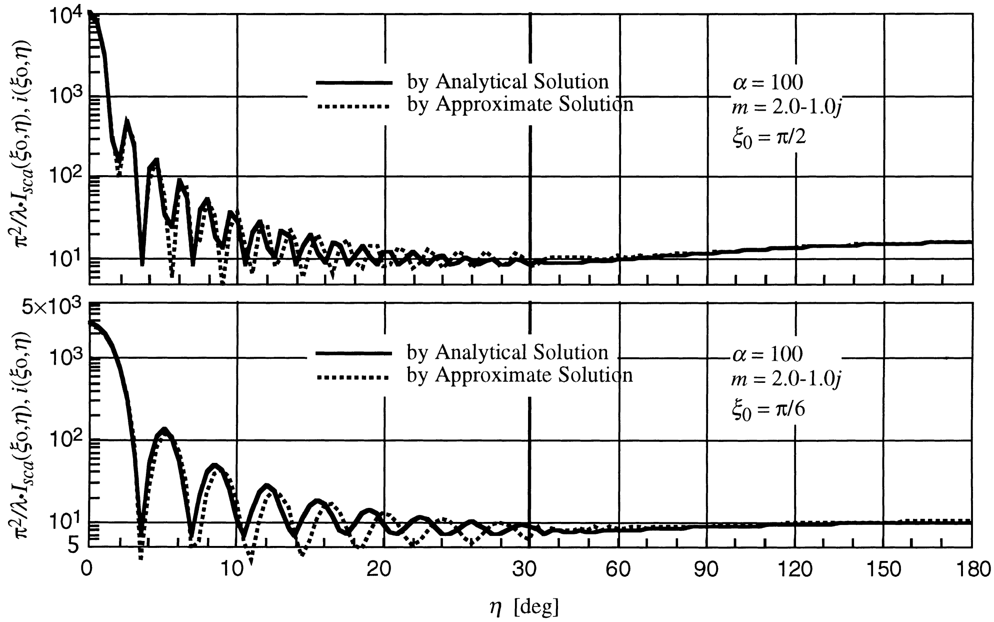

Fig. 5. Energy distribution of radiation scattering from a single fiber.

$$I_{\rm sca}(\xi_0,\eta)=I_{\rm dif}(\xi_0,\eta)+I_{\rm refl}(\xi_0,\eta) \tag{13}$$

while that corresponding to the intensity function is

$$\frac{\pi^2}{\lambda}\frac{I_{\rm sca}(\xi_0,\eta)}{i_0} \tag{14}$$

Fig. 5 shows typical results, where the analytical and approximate solutions for the energy distributions of radiation scattering are in good agreement; thereby indicating that the presented approximation holds for sufficiently large α and κ values.

3.4. Approximate extinction and scattering efficiencies

To determine the range of α and κ in which our approximation can be suitably used, we compared the extinction and scattering efficiencies derived from the approximate scattering energy with those calculated by the analytical solution.

The scattering efficiency $Q_{\rm sca}(m,\alpha,\xi_0)$ and extinction efficiency $Q_{\rm ext}(m,\alpha,\xi_0)$ are respectively denoted in this section as $Q_{\rm sca}(\xi_0)$ and $Q_{\rm ext}(\xi_0)$ or $Q_{\rm sca}$ and $Q_{\rm ext}$ for simplicity.

If $Q_{\rm sca}$ is considered to be the sum of the diffraction and reflection components of the scattering efficiency, then

$$Q_{\rm sca}(\xi_0)=Q_{\rm dif}(\xi_0)+Q_{\rm refl}(\xi_0) \tag{15}$$

where the diffraction component of the scattering efficiency $Q_{\rm dif}$ is

$$Q_{\rm dif}(\xi_0)=\frac{1}{d}\int_{-\pi/2}^{\pi/2}\frac{I_{\rm dif}(\xi_0,\eta)}{i_0}\,{\rm d}\eta \tag{16}$$

and the reflection component $Q_{\rm refl}$ is

$$Q_{\rm refl}(\xi_0)=\frac{1}{d}\int_{-\pi}^{\pi}\frac{I_{\rm refl}(\xi_0,\eta)}{i_0}\,{\rm d}\eta \tag{17}$$

If $Q_{\rm ext}$ is considered to be the sum of the extinction efficiency components of diffraction, reflection, and absorption, then

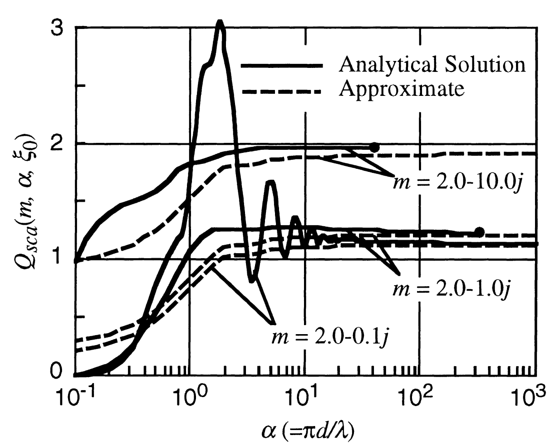

Fig. 6. Comparison of scattering efficiency obtained using the analytical and approximate solutions.

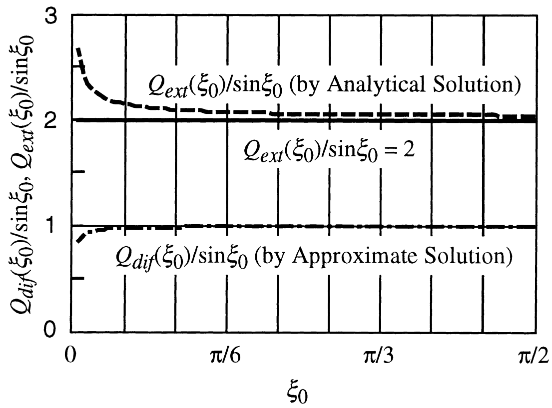

Fig. 7. Effect of the incident angle ξ_0 on the extinction efficiency.

$$Q_{\text{ext}}(\xi_0) = Q_{\text{dif}}(\xi_0) + Q_{\text{refl}}(\xi_0) + Q_{\text{abs}}(\xi_0) \tag{18}$$

where Q_{dif} is given by Eq. (16). The sum of the reflection and absorption components, $Q_{\text{refl}} + Q_{\text{abs}}$, can be derived as shows using the cross sections for reflection and absorption.

Consider the radiation extinction caused only by the reflection and absorption components. The sum of the cross sections by reflection and absorption, $C_{\text{refl}} + C_{\text{abs}}$, which is equal to the projected area of the fibers, $d \sin \xi_0 \cdot$ $Q_{\text{refl}} + Q_{\text{abs}}$ can subsequently be determined after dividing by the fiber diameter d, such that

$$Q_{\text{refl}}(\xi_0) + Q_{\text{abs}}(\xi_0) = \sin \xi_0 \tag{19}$$

These approximations were comparatively evaluated with the analytical results calculated at double precision, as shown in Fig. 6. Note that at $\alpha > 50$ the efficiencies are constant and show good agreement, which indicates Eq. (15) is a good approximation. In addition, at $\alpha > 50$ only the use of a constant value is suitable.

Although Q_{refl} is independent of α, Q_{dif} approaches a constant value as α increases, which in turn causes Q_{sca} to approach a constant value. Numerical analysis results show that the constant value of Q_{dif} at $\alpha > 50$ can be expressed as

$$Q_{\text{dif}}(\xi_0) = \sin \xi_0 \tag{20}$$

Fig. 7 presents numerical results for $Q_{\text{dif}}/\sin \xi_0$ at $\alpha = 100$, where $Q_{\text{dif}}/\sin \xi_0$ is close to unity.

Based on these results, we can express the scattering efficiency as

$$Q_{\text{sca}}(\xi_0) = \sin \xi_0 + Q_{\text{refl}}(\xi_0) \tag{21}$$

where Q_{refl} is determined by Eq. (16).

Since Eq. (20) is also appropriate, we can then derive the following approximate equation for the extinction efficiency Q_{ext} using Eqs. (18), (19) and (20):

$$Q_{\text{ext}}(\xi_0) = 2\sin \xi_0 \tag{22}$$

which is considered to be suitable, as Fig. 7 shows the corresponding results at $\alpha = 100$.

We compared Eqs. (21) and (22), and the corresponding analytical solutions at several m and α values, and found only a 5% difference between them in the regions of $50 < \alpha < 1000$, $0.1 < \kappa < 10$, and $\alpha \cdot \kappa < 500$.

If α or κ are increased higher than the upper limits of these regions, the present approximate efficiencies approach close to those derived by Swathi and Tong's algorithm [7], and consequently, the present approximations are considered to be suitable in the regions of $50 < \alpha$ and $0.1 < \kappa$.

4. Radiative characteristics of fibers having a diffuse surface

We have assumed that the fiber surface is optically smooth. However, as the wavelength shortens, the effect of surface roughness on the radiative transfer cannot be neglected, and reflection off the fiber surface will be more diffuse. In this section, we will describe the radiative characteristics of fibers having a perfectly diffuse surface.

It was described that for large size parameters and large absorption indices, the scattering by a fiber is approximated by superimposition of the diffraction intensity by a rectangular aperture and the specular reflection intensity off the fiber surface. If the approximation can be extended to fibers with a perfectly diffuse surface, we can determine the radiative

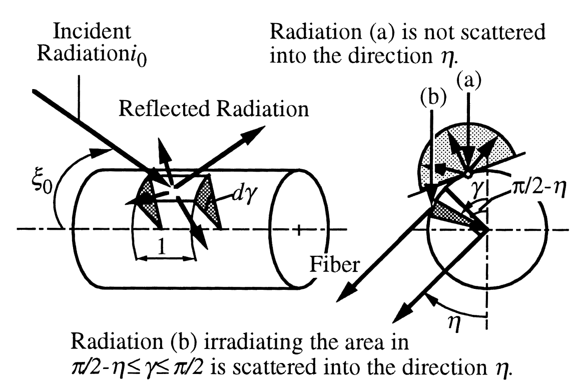

Fig. 8. Diffuse reflection by a fiber surface.

characteristics by superimposing the diffracted radiation intensity and the diffusely reflected one.

4.1. Scattering by diffuse reflection

In case of diffusely reflecting fibers, radiation is reflected in not only the directions of $\xi = \xi_0$, but also in other directions, which is different from the case of specularly reflecting fibers. Fig. 8 shows a diffusely reflecting fiber that is irradiated by the incident radiation in the direction of $\xi = \xi_0$. The reflected energy in the direction ($\xi = \xi, \eta = \eta$) by area $rd\gamma \times 1$ is expressed as

$$\frac{\rho_{\mathrm{d}} \cos\gamma \sin\xi_0 (rd\gamma \times 1) i_0 \cos(\pi - \eta - \gamma) \sin\xi}{\pi} \tag{23}$$

where i_0 is the energy flux of the incident radiation, and ρ_{d} is the diffuse reflectivity off the surface.

The surface area in the ranges of $\pi/2 - \eta < \gamma < \pi/2$ can reflect the radiation in the direction ($\xi = \xi, \eta = \eta$), as shown in Fig. 8. Therefore, the total energy reflected in this direction is given as

$$\begin{aligned} I_{\mathrm{drefl}}(\xi_0, \xi, \eta) &= \frac{\rho_{\mathrm{d}} r \sin\xi_0 \sin\xi \cdot i_0}{\pi} \int_{\pi/2-\eta}^{\pi/2} \cos\gamma \cos(\pi - \eta - \gamma)\, \mathrm{d}\gamma \\ &= \frac{\rho_{\mathrm{d}} r \sin\xi_0 \sin\xi (\sin\eta - \eta\cos\eta) i_0}{4\pi} \end{aligned} \tag{24}$$

Using $I_{\mathrm{drefl}}(\xi_0, \xi, \eta)$, the radiation energy scattered by a single fiber having a diffuse surface is expressed as

$$I_{\mathrm{sca}}(\xi_0, \xi, \eta) = I_{\mathrm{dif}}(\xi_0, \xi, \eta) + I_{\mathrm{drefl}}(\xi_0, \xi, \eta) \tag{25}$$

For specularly reflecting fibers, the scattering can be approximated by superimposition of the diffraction and specular reflection in the ranges of $50 < \alpha$ and $0.1 < \kappa$. For diffusely reflecting fibers, Eq. (25) is considered to be valid in the corresponding ranges of α and κ. However, since the reflected energy is not given by a function of the complex refractive index, but by that of the diffuse reflectivity, we cannot adopt '$0.1 < \kappa$' as the useful range of Eq. (25). Although we cannot give the range of κ, it is considered that Eq. (25) is valid for opaque fibers that absorbs all the radiation energy penetrated into the fiber.

4.2. Extinction and scattering efficiencies of diffusely reflecting fibers

For specularly reflecting fibers, the extinction coefficient can be given by the sum of the diffraction efficiency and the reflection–absorption efficiency. We derive the extinction efficiencies of diffusely reflecting fibers by the same manner as that of fibers having a specular surface.

Because all the radiation energy irradiated onto a fiber surface is absorbed or reflected by the fiber despite the surface characteristics, the reflection–absorption efficiency can be written as in Eq. (19). The diffraction efficiency of fibers does not depend on the surface characteristics and is given by Eq. (20), Therefore, the extinction efficiency of the diffusely reflecting fibers is also given by Eq. (22)

In order to determine the scattering efficiency, we need to obtain the diffuse reflection efficiency. The diffuse reflection efficiency, Q_{drefl}, is defined as the total reflected radiation energy divided by the incident energy. Using Eq. (24) for the directional reflected radiation energy, Q_{drefl}, is given as

$$\begin{aligned} Q_{\mathrm{drefl}}(\xi_0) &= \frac{1}{d} \int_{-\pi}^{\pi} \int_0^{2\pi} \frac{I_{\mathrm{drefl}}(\xi_0, \xi, \eta)}{i_0} \sin\xi \, \mathrm{d}\xi \mathrm{d}\eta \\ &= \rho_{\mathrm{d}} \sin\xi_0 \end{aligned} \tag{26}$$

Summing Q_{drefl} of Eq. (26) and Q_{dif} of Eq. (20), the scattering efficiency of diffuse reflecting fibers is

$$Q_{\mathrm{sca}}(\xi_0) = \left(1 + \rho_{\mathrm{d}}\right) \sin\xi_0 \tag{27}$$

5. Radiative properties in fibrous media

The radiative properties of fibrous media, such as extinction and scattering coefficients, $\beta(\boldsymbol{\Omega}')$ and $\sigma_{\mathrm{s}}(\boldsymbol{\Omega}')$ and phase function $\Phi(\boldsymbol{\Omega}', \boldsymbol{\Omega})$, can be derived by using the radiative characteristics of the fibers. According to the literature [6], the radiative properties are expressed as

$$\beta(\boldsymbol{\Omega}') = \frac{4f_v}{\pi d} \oint_{R_{\mathrm{f}}=2\pi} Q_{\mathrm{ext}}\left(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}\right) P(\boldsymbol{R}_{\mathrm{f}})\, \mathrm{d}R_{\mathrm{f}} \tag{28}$$

$$\sigma_{\mathrm{s}}(\boldsymbol{\Omega}') = \frac{4f_v}{\pi d} \oint_{R_{\mathrm{f}}=2\pi} Q_{sca}\left(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}\right) P(\boldsymbol{R}_{\mathrm{f}})\, \mathrm{d}R_{\mathrm{f}} \tag{29}$$

and

$$\Phi\left(\boldsymbol{\Omega}', \boldsymbol{\Omega}\right) = \frac{\oint_{R_{\mathrm{f}}=2\pi} Q_{\mathrm{sca}}\left(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}\right) p_{\mathrm{f}}\left(\boldsymbol{\Omega}', \boldsymbol{\Omega}, \boldsymbol{R}_{\mathrm{f}}\right) P(\boldsymbol{R}_{\mathrm{f}})\, \mathrm{d}R_{\mathrm{f}}}{\oint_{R_{\mathrm{f}}=2\pi} Q_{\mathrm{sca}}\left(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}\right) P(\boldsymbol{R}_{\mathrm{f}})\, \mathrm{d}R_{\mathrm{f}}} \tag{30}$$

where $\boldsymbol{\Omega}', \boldsymbol{\Omega}$ and $\boldsymbol{R}_{\mathrm{f}}$ are respectively the unit vectors for the direction of the incident radiation, scattered radiation, and the fiber, while $P(\boldsymbol{R}_{\mathrm{f}})$ is the density function representing the fiber orientation defined by

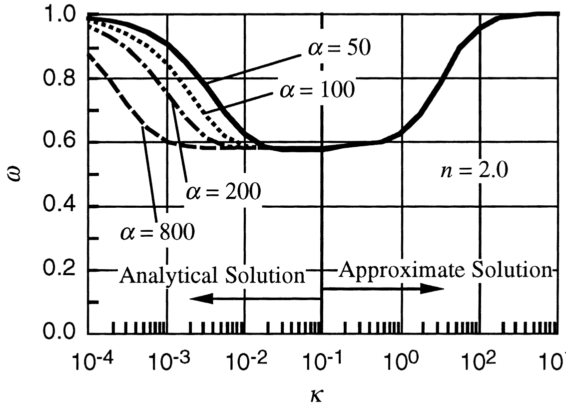

Fig. 9. Effect of the absorption index on the albedo.

Komori and Makishima [11], and p_f is the scattering phase function of a single fiber (for details see Yamada and Kurosaki [6]).

Here, radiation transfer is considered for fibrous media where all fibers in the media are randomly oriented in space. The density function of the media is expressed as

$$P(\boldsymbol{R}_f) = \frac{1}{2\pi} \tag{31}$$

Using Eqs. (28)–(31), we can clarify the effects of the radiative characteristics of fibers on the radiative properties of fibrous media.

5.1. Extinction coefficients of fibrous media

Since the extinction efficiencies of fibers with a large size parameter and a large absorption index are the same despite the fibers' surface characteristics, smooth or rough, the extinction coefficient can be expressed in simple form using Eqs. (22), (28) and (31), i.e.,

$$\beta = \frac{4f_v}{\pi d}\frac{\pi}{2} \tag{32}$$

5.2. Albedo of fibrous media

The albedo of media is defined as

$$\omega(\boldsymbol{\Omega}') = \frac{\sigma_s(\boldsymbol{\Omega}')}{\beta(\boldsymbol{\Omega}')} \tag{33}$$

Since the scattering and extinction efficiencies are given by Eqs. (21), (22) and (27), the albedo can be calculated using Eqs. (28), (29) and (33). For the media with diffusely reflecting fibers, the albedo can be expressed in simple form as

$$\omega = \frac{1+\rho_d}{2} \tag{34}$$

Fig. 9 shows the albedo for media with specularly reflecting fibers with respect to the absorption index of the fibers. The analytical solution was used for calculating the albedo for media with fibers having $\kappa \leq 0.1$, and the present approximate solution was used for those having $0.1 < \kappa$. As shown in Fig. 9, when κ is small, the radiation penetrating into a fiber is attenuated by absorption, and therefore the albedo decreases and reaches a minimum value with increasing κ. In contrast, when κ is large, the albedo increases with increasing κ because more radiation is reflected by the fiber surface.

Fig. 9 also shows that the albedo is small in media with large α, even if the absorption index of the fibers is comparatively small. This is because more radiation is absorbed in the fibers as α increases.

5.3. Scattering phase functions of fibrous media

To compute the scattering phase function using Eq. (25), a special technique for integrating the numerator is required. The scattered (diffracted or specularly reflected) radiation by a fiber with a smooth surface propagates into the direction, $\xi = \xi_0$, i.e., the Dirac delta function is included in the scattering phase function of the fiber, $p_f(\boldsymbol{\Omega}', \boldsymbol{\Omega}, \boldsymbol{R}_f)$. The computing technique for the medium with all fibers randomly oriented in space is shown by Houston and Korpela [4].

For diffusely reflecting fibers, although the diffracted radiation propagates into the direction, $\xi = \xi_0$, the reflected radiation propagates into all directions. In this case, the special technique by Houston and Korpela cannot be adopted for the reflected radiation. Therefore, the total scattering phase function, including the diffracted and reflected components, is determined from those by diffraction and reflection, which are individually calculated by different computational techniques.

The scattering phase functions by diffraction and reflection are defined as, respectively

$$\Phi_{\mathrm{dif}}(\boldsymbol{\Omega}', \boldsymbol{\Omega}) = \frac{\oint_{R_f=2\pi} Q_{\mathrm{dif}}(\boldsymbol{\Omega}', \boldsymbol{R}_f) p_{f,\mathrm{dif}}(\boldsymbol{\Omega}', \boldsymbol{\Omega}, \boldsymbol{R}_f) P(\boldsymbol{R}_f)\, \mathrm{d}R_f}{\oint_{R_f=2\pi} Q_{\mathrm{dif}}(\boldsymbol{\Omega}', \boldsymbol{R}_f) P(\boldsymbol{R}_f)\, \mathrm{d}R_f} \tag{35}$$

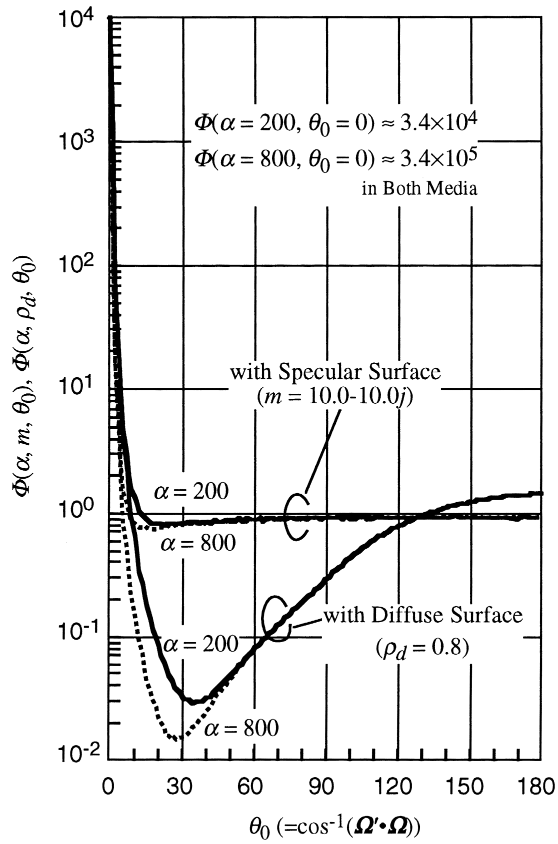

Fig. 10. Scattering phase functions of media with randomly oriented fibers.

$$\Phi_{\mathrm{drefl}}(\boldsymbol{\Omega}', \boldsymbol{\Omega}) = \frac{\oint_{R_{\mathrm{f}}=2\pi} Q_{\mathrm{dref}}(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}) p_{\mathrm{f,drefl}}(\boldsymbol{\Omega}', \boldsymbol{\Omega}, \boldsymbol{R}_{\mathrm{f}}) P(\boldsymbol{R}_{\mathrm{f}})\, \mathrm{d}R_{\mathrm{f}}}{\oint_{R_{\mathrm{f}}=2\pi} Q_{\mathrm{dref}}(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}) P(\boldsymbol{R}_{\mathrm{f}})\, \mathrm{d}R_{\mathrm{f}}} \tag{36}$$

The scattering phase function by diffraction can be computed by Houston and Korpela's technique. The scattering phase function by diffuse reflection can be calculated by the usual technique for numerical integration because the scattering phase function of a diffusely reflecting fiber, $p_{\mathrm{f}}(\boldsymbol{\Omega}', \boldsymbol{\Omega}, \boldsymbol{R}_{\mathrm{f}})$ in Eq. (30) does not include the Dirac's delta function.

From Eqs. (20), (26) and (27), the following relationships are derived.

$$Q_{\mathrm{sca}}(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}) = (1 + \rho_{\mathrm{d}}) Q_{\mathrm{dif}}(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}) \tag{37}$$

$$Q_{\mathrm{sca}}(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}) = \frac{(1 + \rho_{\mathrm{d}})}{\rho_{\mathrm{d}}} Q_{\mathrm{drefl}}(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}) \tag{38}$$

Using these equations, the scattering phase function of the media with diffusely reflecting fibers can be expressed as

$$\begin{aligned}\Phi(\boldsymbol{\Omega}', \boldsymbol{\Omega}) &= \frac{\oint_{R_{\mathrm{f}}=2\pi} Q_{\mathrm{dif}}(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}) p_{\mathrm{f,dif}}(\boldsymbol{\Omega}', \boldsymbol{\Omega}, \boldsymbol{R}_{\mathrm{f}}) P(\boldsymbol{R}_{\mathrm{f}})\, \mathrm{d}R_{\mathrm{f}}}{\oint_{R_{\mathrm{f}}=2\pi} Q_{\mathrm{sca}}(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}) P(\boldsymbol{R}_{\mathrm{f}})\, \mathrm{d}R_{\mathrm{f}}} \\ &+ \frac{\oint_{R_{\mathrm{f}}=2\pi} Q_{\mathrm{drefl}}(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}) p_{\mathrm{f,drefl}}(\boldsymbol{\Omega}', \boldsymbol{\Omega}, \boldsymbol{R}_{\mathrm{f}}) P(\boldsymbol{R}_{\mathrm{f}})\, \mathrm{d}R_{\mathrm{f}}}{\oint_{R_{\mathrm{f}}=2\pi} Q_{\mathrm{sca}}(\boldsymbol{\Omega}', \boldsymbol{R}_{\mathrm{f}}) P(\boldsymbol{R}_{\mathrm{f}})\, \mathrm{d}R_{\mathrm{f}}} \\ &= \frac{1}{(1 + \rho_{\mathrm{d}})} \Phi_{\mathrm{dif}}(\boldsymbol{\Omega}', \boldsymbol{\Omega}) + \frac{\rho_{\mathrm{d}}}{(1 + \rho_{\mathrm{d}})} \Phi_{\mathrm{drefl}}(\boldsymbol{\Omega}', \boldsymbol{\Omega})\end{aligned} \tag{39}$$

Fig. 10 shows the scattering phase functions for media with specularly reflecting fibers and those with diffusely reflecting fibers. The complex refractive index of the specularly reflecting fibers is $m = 10.0 - 10.0j$ and the reflectivity of the diffuse reflecting fibers is $\rho_{\mathrm{d}} = 0.8$. Since all fibers are randomly oriented in space in these media, the scattering phase function becomes a function of the angle between the incident and scattering directions, $\theta_0\,(= \cos^{-1}(\boldsymbol{\Omega}' \cdot \boldsymbol{\Omega}))$. The phase functions are plotted against θ_0 in Fig. 10.

The strong forward scattering close to $\theta_0 = 0$ is caused by the diffraction component and the weak scattering in the other θ_0 is by the reflection component. This figure shows that the forward scattering of the media with $\alpha = 800$ is steeper than those with $\alpha = 200$. This is because diffraction becomes steeper with increasing size parameters. This figure also shows that the backward scattering of media with diffusely reflecting fibers is larger than that of specularly reflecting fibers.

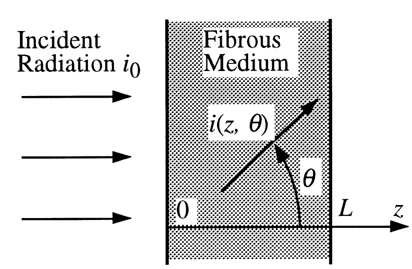

Fig. 11. Coordinates system in a planar fibrous medium.

6. Radiation transfer in planar fibrous media

Let us consider radiation transfer in planar fibrous media that is irradiated normal to their boundaries as shown in Fig. 11. The radiation transfer is therefore azimuthally symmetric. Supposing that internal emission is negligible, the governing equation for radiation intensity $i(z, \theta)$ is

$$\cos\theta\frac{\mathrm{d}i(z,\theta)}{\mathrm{d}z}+\beta\cdot i(z,\theta)=\frac{\sigma_s}{2}\int_0^{\pi}\bar{\Phi}(\theta',\theta)i(z,\theta')\sin\theta'\,\mathrm{d}\theta' + \frac{\sigma_s}{4\pi}\bar{\Phi}(\theta'=0,\theta)i_0\exp(-\beta\cdot z) \quad (40)$$

with the boundary conditions being

$$i(z=0, 0\le\theta<\pi/2)=0 \quad (41)$$

$$i(z=L, -\pi/2<\theta\le 0)=0 \quad (42)$$

where L is the thickness of the medium, and i_0 the heat flux of the irradiation.

β and σ_s are independent of the radiation propagation direction because all fibers are randomly oriented in this medium. $\bar{\Phi}(\theta', \theta)$ is the averaged phase function over all azimuthal angles, i.e.,

$$\bar{\Phi}(\theta',\theta)=\frac{1}{2\pi}\int_0^{2\pi}\Phi(\theta',\theta,\phi_0)\,\mathrm{d}\phi_0 \quad (43)$$

where $\Phi(\theta', \theta, \phi_0)$ is the original phase function expressed using the medium coordinates, while ϕ_0 is the difference between the azimuthal angle of the direction of incidence and the scattered radiation [12].

In this analysis, we discuss the effect of surface characteristics, specular or diffuse, on the radiation transfer in fibrous media. It is possible to determine the effect of different surface characteristics of fibers on the radiation transfer, regardless of whether or not the fibers have the same diameter and/or material. However, it should be noted that for a specularly reflecting fiber, the radiative characteristics are given by a function of the complex refractive index, whereas for a diffusely reflecting fiber, they are given by a function of the diffuse reflectivity. Therefore, it is difficult to determine the effect only by the surface characteristics.

Here, we have selected the values of the complex refractive index and the diffuse reflectivity of fibers so that the values of the albedo in both media coincide with each other. Given the same diameter, we have clarified the effect of only the scattering phase functions, which are different from each other due to their surface characteristics. We selected $m = 10.0 - 10.0j$ for the complex refractive index of the fiber with a specular surface, and $\rho_d = 0.8$ for the diffuse reflectivity of the fiber with a diffuse surface. The values of the albedo become 0.9 in each media.

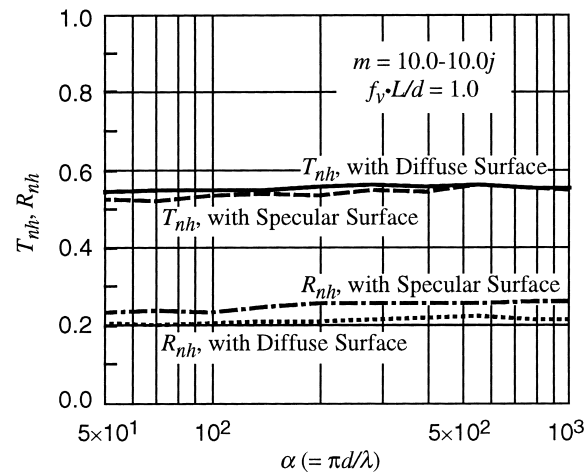

Fig. 12. Radiation transfer in planar media with randomly oriented fibers.

Eq. (32) was numerically solved using the discrete ordinates method. The results are shown in Fig. 12, where the hemispherical transmittance T_{nh} and the hemispherical reflectance R_{nh} are plotted against α. The increasing of α when $f_v \times L/d$ is constant indicates that the wavelength of the irradiation upon the fibrous medium decreases in inverse proportion to α.

Fig. 12 shows that T_{nh} and R_{nh} are weakly dependent on α (when $f_v \times L/d$ is constant, α can be regarded as $1/\lambda$). This implies that the radiative characteristics are almost independent of α when the radiative properties of fibers in the media does not depend on the wavelength.

When an increase of α implies an increase of d, e.g., $f_v \times L/\lambda$ is constant, α would strongly affect the radiation transfer, because the extinction coefficients of the media are proportional to d.

The difference between T_{nh} of both fibrous media cannot be seen, whereas R_{nh} of the media with specularly reflecting fibers is 10–30% larger than that of the diffusely reflecting one. This is caused by the stronger backward scattering of the media with diffusely reflecting fibers, as shown in Fig. 10.

It should be noted that, when the fibers in both media are made of the same material, R_{nh} of the media with diffusely reflecting fibers are not always larger. The results shown in Fig. 12 are derived assuming that the albedo values of both media are equal. If the fibers in both media were made of the same material, the albedo of the media with diffusely reflecting fibers would be smaller than that of the specularly reflecting fibers due to the multi-reflection on the surface. In this case, R_{nh} and T_{nh} of the media with fibers having a diffuse surface would be smaller.

7. Conclusions

This study shows that the scattering intensity by a single fiber having a large size parameter and a large absorption index can be approximated by superimposition of the diffraction intensity by a rectangular aperture and the specular reflection intensity off the fiber surface. The ranges of size parameters and absorption indices where the present approximation is valid have been clarified. By extending this approximation, we predicted the radiative characteristics of fibers having a diffuse surface.

Moreover, by using this approximation, we are able to estimate the radiative properties of media where fibers with a large size parameter are randomly oriented, and investigate the radiation transfer in planar media having randomly oriented fibers.

As a result, if the size parameter is sufficiently large, the fibrous media will have small albedo even if they have comparatively small absorption indices. The scattering phase functions show that the media with diffusely reflecting fibers have a larger backward scattering.

It is found that when the fiber diameter is fixed the effect of the size parameter on the radiation transfer is weak for planar media irradiated by collimated radiation, and that the difference in the scattering phase function caused by the surface characteristics of the fibers does not greatly affect the hemispherical transmittance but rather affect the hemispherical reflectance.

Appendix

Radiation energy diffracted by a rectangular aperture is derived from an equation applied to consider electric field intensity. Therefore, if we assume that E_0 represents the electric field intensity of a light source which is located at distance l_0 from the aperture, then the electric field intensity at aperture E_a can be expressed as

$$E_a = (E_0/l_0)\exp(-jkl_0) \tag{A1}$$

where k is the wave number. Now, let E_p be electric field intensity of the diffracted radiation at distance r along the direction ξ, η, i.e,

$$E_p = AGbd\frac{\sin\phi}{\phi}\frac{\sin\psi}{\psi} \tag{A2}$$

where

$$A = -\frac{jE_0}{\lambda l_0 r}\exp\{-jk(l_0 + r)\} \tag{A3}$$

with G, ϕ and ψ are given in Eqs. (4)–(6) [13].

Since radiation energy is proportional to the squared electric field intensity, the irradiation energy i_0 can be expressed using a proportional constant C and Eq. (A1), such that

$$i_0 = C|E_a|^2 \tag{A4}$$

and from Eq. (A2), the diffracted radiation energy per unit solid angle, i'_{aper}, in the direction of (ξ, η) is obtained by multiplying by r^2, i.e.,

$$i'_{aper} = C|E_p|^2 r^2 \tag{A5}$$

By eliminating C from Eqs. (A4) and (A5), the diffracted radiation energy can be derived as Eq. (3).

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